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Non-BPS Branes in a Type I Orbifold

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Abstract

We analyse the spectrum of non-BPS branes in the type I theory on the orbifold $T^4/\mathcal{I}_4$. We present a detailed analysis of the action of the worldsheet parity Ω on the different D-brane boundary state sectors of type IIB on $T^4/\mathcal{I}_4$, using the covariant formulation. Using these results we derive the spectrum of branes in the type I orbifold. We find $\mathbb{Z}_2$- and $\mathbb{Z}$-charged non-BPS branes. A study of the stability of these branes in the type I orbifold is also presented. Among other stable branes, the theory contains a stable non-BPS D-instanton which carries twisted R-R charge.

Contents

1 Introduction

In recent years, significant progress has been made towards understanding the various string duality symmetries. The validity of such dualities has often lead to new findings in string theory. For example, the strong-weak coupling duality between $SO(32)$ heterotic string and the type I theory has compelled to look for the states in type I theory which are the dual partners of stable but non-BPS states in the heterotic string theory. A description of such states in type I theory is found to be in terms of a tachyon kink solution on a D-brane anti-D-brane pair [1, 2, 3]. Subsequently, this phenomena and other duality relations have given rise to a tremendous activity in constructing and understanding the dynamics of non-BPS branes in string theories [1].

The present understanding is that type IIA (IIB) theory admits Dp-branes which are BPS for p even (odd) and non-BPS for p odd (even) for all values of p ranging from zero to nine. Furthermore, in both theories a descent relation has been established [5] involving BPS and non-BPS branes, i.e. a Dp-brane is seen as a descendant of a D$(p+1)$-brane. Moreover, one can obtain a Dp-brane of IIA (IIB) from a Dp-brane of IIB (IIA) by modding out with the discrete symmetry $(-1)^{F_L}$. BPS branes preserve

[1] For reviews on non-BPS branes see [4]

half of the spacetime supersymmetries and are stable, whereas non-BPS branes break all the spacetime supersymmetries and are unstable. This instability can be explained by the presence of a tachyon in the open string spectrum of the brane. The effective action for these non-BPS branes, including the tachyon field, has been studied in [6]. Recently, a background independent effective action has been constructed for the case of non-commutative branes [7].

The unstable non-BPS branes of type II theories can decay to the stable BPS branes via the condensation of a solitonic configuration of the tachyon field. This tachyon condensation has been extensively studied using string field theory [8] and p-adic string theory [9]. In the framework of non-commutative field theory [10] the above scenario is also understood via non-commutative solitons [11].

Although much of the above is understood in type II theories, less is known for non-BPS branes in type I theory, specially in orbifold backgrounds. The most interesting feature of the non-BPS branes in orbifold and orientifold theories is that they may be stable [1, 2, 3, 12, 13, 14]. This stability appears because of the fact that the tachyon state is projected from the open string spectrum on the brane by the orbifold/orientifold symmetry.

In this paper, we study the spectrum of fractional and non-BPS branes in type I theory on a $T^4/\mathcal{I}_4$ orbifold. This orbifold is equivalent to the orientifold by the worldsheet parity-reversal Ω of type IIB on $T^4/\mathcal{I}_4$. Consistency of this type I orbifold implies that the theory must contain D5 and D9 branes, with unitary and symplectic subgroups of $U(16)\times U(16)$ [15, 16, 17, 18, 19].

We follow the notation of [20] for D-branes where a Dp-brane in the orbifold $T^4/\mathcal{I}_4$ of a type II theory is denoted by $|D(r,s)\rangle$, $r+s=p$, where r denotes the number of spatial Neumann directions along the fixed plane and s denotes those along the orbifolded directions[2].

We describe D-branes using the boundary state formalism, where D-branes are described by physical closed string states of the bosonic spectrum. In orbifold theories there will be twisted sectors as well and the D-branes may also contain boundary states in these sectors. Following [20], in type II theories orbifolded by $\mathcal{I}_4$ (or $(-1)^{F_L}\cdot\mathcal{I}_4$) one finds BPS branes that can be either fractional or bulk branes. Fractional branes in type IIB on $T^4/\mathcal{I}_4$ can exist for r odd and s even, and have boundary states in all sectors of the theory[3]:

$$\begin{aligned} |D(r,s)\rangle_f \; &= \; |D(r,s)\rangle_{\mathrm{NS,U}} + \epsilon_1\, |D(r,s)\rangle_{\mathrm{R,U}} \\ &\quad + \epsilon_2 \sum_{a=1}^{2^s} \mathrm{e}^{i\theta_a} \left(|D(r,s)\rangle_{\mathrm{NS,T_a}} + \epsilon_1 |D(r,s)\rangle_{\mathrm{R,T_a}}\right) . \end{aligned} \tag{1.1}$$

[2]For simplicity, for a $|D(r,s)\rangle$ and, unless otherwise stated, we will always choose the s Neumann directions starting from x^6; so that, for instance, a $D(1,2)$ is wrapped around the directions x^6 and x^7 of the T^4.

[3]For short we use NS and R instead of NS-NS and R-R, respectively, as subindices in the boundary states.

Each twisted sector is located in one of the 16 orbifold fixed planes. Furthermore, each boundary state sector is given by a GSO-invariant combination of boundary states.

Bulk branes can be described as branes with only untwisted sectors:

$$|D(r,s)\rangle_b = |D(r,s)\rangle_{\rm NS,U} + |D(r,s)\rangle_{\rm R,U} \, . \tag{1.2}$$

In general for type II orbifold they exist for the same value as the fractional branes do, since two fractional branes with opposite twisted charges can join and give rise to a bulk brane. This brane can then move off the fixed planes.

Non-BPS branes in type II $\mathbb{Z}_2$ orbifolds are referred to as truncated branes and are represented as follows:

$$|D(r,s)\rangle_t = |D(r,s)\rangle_{\rm NS,U} + \epsilon \sum_{a=1}^{2^s} {\rm e}^{i\theta_a} \, |D(r,s)\rangle_{\rm R,T} \, . \tag{1.3}$$

In type IIB on $T^4/\mathbb{Z}_2$ they exist for r and s odd [20]. They carry twisted R-R charge only, so they are referred to as $\mathbb{Z}$-charge non-BPS branes. In the conventions of this paper, these branes have a tension given by:

$$T_{(r,s)} = (\alpha')^{\frac{2-p}{2}} (2\pi)^{3-p} \sqrt{\pi} \, , \qquad r+s=p \, , \tag{1.4}$$

which coincides with the tension of a BPS Dp-brane in type IIA; and a (twisted R-R) charge

$$\tilde{Q}_{(r,s)} = (2\pi\sqrt{\alpha'})^{3-r} \pi^{-\frac{3}{2}} (\alpha')^{-1} \, , \tag{1.5}$$

which does not depend on the number of Neumann directions along the orbifolded 4-torus.

On the other hand, BPS branes in type I theory are given in terms of boundary states as in type II with the addition of a 9-crosscap[4]:

$$|Dp\rangle = \frac{1}{\sqrt{2}} \left(|Dp\rangle_{\rm NS} + |Dp\rangle_{\rm R} + |C9\rangle \right) \, . \tag{1.6}$$

The 9-crosscap is the closed string description of the orientifold 9-plane. BPS branes exist for $p = 1, 5, 9$, which are the values of p for which the corresponding R-R potential survives the Ω-projection.

Non-BPS branes in type I are given by just an NS-NS boundary state plus the crosscap contribution:

$$|Dp\rangle = \frac{1}{\sqrt{2}} \left(|Dp\rangle_{\rm NS} + |C9\rangle \right) \, . \tag{1.7}$$

For $p = -1, 0, 7$ and 8 the contribution from the crosscap is such that the tachyons on the worldvolume of the non-BPS branes are projected out and thus giving the possibility of having stable non-BPS D-branes in type I [3, 13, 14].

[4] Here, and in subsequent formulae, the background D9 (and eventually the D5) branes are not explicitly displayed, but are understood.

The theory we consider in this article is type I on $T^4/\mathcal{I}_4$, which can be seen either as an orbifold of type I or as an orientifold of the type IIB orbifold. Therefore, the spectrum of D-branes of the type I orbifold can be deduced by either applying the orbifold projection on the type I D-branes or applying the Ω projection to the D-branes of the type IIB orbifold. For the first case we must take into account as well the twisted sectors. Our approach consists in first studying the Ω projection on the boundary states of the type IIB orbifold, for which we use the covariant formulation of the boundary states [21]. This analysis has not been performed before in the literature[5] and clarifies interesting subtleties about the action of Ω on the boundary states. Adding up the information about the $\mathcal{I}_4$ projection on the boundary states given in [20], we can deduce the boundary states which will survive in the type I orbifold. The next step is to put together these states to make up the D-brane states of the theory. By the nature of the orbifold, we can deduce that there are bulk and fractional BPS D-branes. Regarding the non-BPS branes, some of them are truncated with twisted R-R charge, similar to the type IIB orbifold. The rest of them have no R-R charge at all, like in the type I theory. This latter type of brane can be seen as truncated branes of the type IIB orbifold that get their R-R sector projected out by Ω, or else, as non-BPS branes of type I that are not modified by the orbifold. On the other hand, those truncated branes of the type I orbifold that have twisted R-R charge can be seen either as truncated branes of the type IIB orbifold that are not modified by the Ω projection, or as non-BPS branes of type I that receive a contribution from the twisted sector.

The paper is organised as follows. In section 2 we present a thorough study of the action of Ω in the different boundary state sectors. A summary of the results can be found in Table 1. Using these results, in section 3 we give a classification of the BPS and non-BPS branes of the type I orbifold. This classification is summarised in Tables 2 and 3. Section 4 is concerned about the stability of the non-BPS branes in the type I orbifold. In section 5 we discuss the conclusions of our analysis. We include three appendices. Appendix A explains in detail the action of the operator Ω on states in the asymmetric superghost picture $(-1/2, -3/2)$. Appendix B deals with the details of the action of Ω on the twisted NS-NS sector. Finally, Appendix C contains details about the form of the crosscaps states used for the computations of this paper.

2 Action of Ω on the Boundary States

We consider first the action of Ω on the open string sectors. From the mode expansion of the fields we have for the oscillators

$$\begin{aligned} \Omega \alpha_n^\mu \Omega^{-1} &= \pm e^{in\pi} \alpha_n^\mu \,, \\ \Omega \psi_m^\mu \Omega^{-1} &= \pm e^{im\pi} \psi_m^\mu \,, \end{aligned} \tag{2.1}$$

[5] The action Ω in the light-cone gauge has been studied for boundary states in [22], and for string states in [19].

where the plus sign is for the NN directions and the minus sign is for the DD directions. Moreover, Ω relates the DN and ND strings so there is no definite action in these oscillators for these directions. Furthermore, for the NS-vacuum in the (-1) picture we have

$$\Omega|0\rangle_{-1} = -i|0\rangle_{-1}\,, \tag{2.2}$$

whereas the R-vacuum in the $(-1/2)$ picture transforms as:

$$\Omega|a\rangle_{-1/2} = -\Gamma^{\nu_{p+1}}\cdots\Gamma^{\nu_{9-p}}|a\rangle_{-1/2}\,, \tag{2.3}$$

where $\nu_{p+1},\ldots\nu_{9-p}$ are the DD directions.

Regarding the oscillators of the closed string, for convenience, we define Ω as a combination of worldsheet parity-reversal ($\sigma \to 2\pi - \sigma$) and the GSO-projection. With this definition it has the same action on the physical states as worldsheet parity-reversal only. On the oscillators it acts by simply exchanging left and right sectors:

$$\Omega\alpha_n\Omega^{-1} = \tilde{\alpha_n}\,, \qquad \Omega\psi_n\Omega^{-1} = \tilde{\psi_n}\,, \tag{2.4}$$

and analogously for the right sector.

Similarly, Ω exchanges the left and right sectors of the (super)ghosts:

$$\begin{aligned} \Omega b_n\Omega^{-1} &= \tilde{b}_n \quad , \qquad \Omega c_n\Omega^{-1} = \tilde{c}_n\,, \\ \Omega\beta_m\Omega^{-1} &= \tilde{\beta}_m \quad , \qquad \Omega\gamma_m\Omega^{-1} = \tilde{\gamma}_m\,, \end{aligned} \tag{2.5}$$

and similarly for the right sector. The action of Ω on the NS-NS and R-R ground states for untwisted and twisted sectors are studied below, and the results are given in equations (2.7), (2.16), (2.34) and (A.19)

2.1 The Untwisted NS-NS Sector

The untwisted NS-NS boundary state[6]

$$|D(r,s),\eta\rangle_{\mathrm{NS,U}} = \frac{T_{(r,s)}}{2}\,|D(r,s)_X\rangle|D(r,s)_\psi,\eta\rangle_{\mathrm{NS,U}}|D(r,s)_{gh}\rangle|D(r,s)_{sgh},\eta\rangle_{\mathrm{NS}}\,, \tag{2.6}$$

is constructed upon the NS-NS ground state in the $(-1,-1)$ picture. The action of Ω on this ground state is given by:

$$\Omega\left(|0\rangle_{-1}\otimes\widetilde{|0\rangle}_{-1}\right) = \widetilde{|0\rangle}_{-1}\otimes|0\rangle_{-1} = -|0\rangle_{-1}\otimes\widetilde{|0\rangle}_{-1}\,. \tag{2.7}$$

On the other hand, since Ω exchanges left and right oscillators, the overall effect is a change from η to $-\eta$, hence we find:

$$\Omega|D(r,s),\eta\rangle_{\mathrm{NS,U}} = -|D(r,s),-\eta\rangle_{\mathrm{NS,U}}\,. \tag{2.8}$$

Therefore, the GSO projected untwisted NS-NS boundary state

$$|D(r,s)\rangle_{\mathrm{NS,U}} = \frac{1}{2}\left(|D(r,s),+\rangle_{\mathrm{NS,U}} - |D(r,s),-\rangle_{\mathrm{NS,U}}\right)\,, \tag{2.9}$$

is invariant under Ω, for any $p = r + s$.

[6] We use the conventions of [23] for the definition of the boundary states. See also [24] for details.

2.2 The Untwisted R-R Sector

Consider now the untwisted R-R part. The R-R ground state in the $(-1/2,-1/2)$ picture has the following transformation property under Ω:

$$\Omega\left(|A\rangle_{-1/2}\otimes\widetilde{|B\rangle}_{-1/2}\right)=\widetilde{|A\rangle}_{-1/2}\otimes|B\rangle_{-1/2}=-|B\rangle_{-1/2}\otimes\widetilde{|A\rangle}_{-1/2}\,. \tag{2.10}$$

On the other hand, the R-R boundary state in the covariant formalism is most conveniently written in the asymmetric superghost picture $(-1/2,-3/2)$ [25]:

$$|D(r,s),\eta\rangle_{\rm R,U}=\frac{Q_{(r,s)}}{2}\,|D(r,s)_X\rangle|D(r,s)_\psi,\eta\rangle_{\rm R,U}|D(r,s)_{gh}\rangle|D(r,s)_{sgh},\eta\rangle_{\rm R}\,, \tag{2.11}$$

where $r+s=p$ odd in type IIB. The fermionic matter and the superghost components have zero-mode contributions

$$|D(r,s),\eta\rangle^{(0)}_{\rm R,U}=|D(r,s)_\psi,\eta\rangle^{(0)}_{\rm R,U}|D(r,s)_{sgh},\eta\rangle^{(0)}_{\rm R}\,, \tag{2.12}$$

given by

$$\begin{aligned}|D(r,s),\eta\rangle^{(0)}_{\rm R,U}&=\mathrm{e}^{i\eta\gamma_0\tilde\beta_0}\mathcal{M}_{AB}\,|A\rangle_{-1/2}\otimes\widetilde{|B\rangle}_{-3/2}\,,\\ \mathcal{M}_{AB}(\eta)&=\left(\mathcal{C}_{(10)}\Gamma^0\cdots\Gamma^p\frac{1+i\eta\Gamma_{11}}{1+i\eta}\right)_{AB}\,,\qquad p=r+s\,,\end{aligned} \tag{2.13}$$

where A,B are spinor indices of $SO(1,9)$ and $\mathcal{C}_{(10)}$ is the charge conjugation matrix in the corresponding representation of the 10-dimensional Γ-matrices.

Note that under the action of Ω, the superghost pictures seem to be swapped between left and right:

$$\Omega\left(|A\rangle_{-1/2}\otimes\widetilde{|B\rangle}_{-3/2}\right)=\widetilde{|A\rangle}_{-1/2}\otimes|B\rangle_{-3/2}=-|B\rangle_{-3/2}\otimes\widetilde{|A\rangle}_{-1/2}\,. \tag{2.14}$$

The commutation of the superghosts in order to recover the $(-1/2,-3/2)$ could yield a phase that we must calculate. In Appendix A we give the details about how to obtain the action of Ω in the asymmetric picture. Here we present directly the result for the superghost sector:

$$\Omega|D(r,s)_{sgh},\eta\rangle_{\rm R}=-i\eta|D(r,s)_{sgh},-\eta\rangle_{\rm R}\,. \tag{2.15}$$

Regarding the fermion ground state we have

$$\Omega\left(|A\rangle\otimes\widetilde{|B\rangle}\right)=-|B\rangle\otimes\widetilde{|A\rangle}\,, \tag{2.16}$$

hence the effect of Ω on the matrix $\mathcal{M}_{AB}$ is a transposition:

$$\begin{aligned}\mathcal{M}(\eta)^T&=(-1)^{p+\frac{1}{2}p(p+1)}\left(\mathcal{C}\Gamma^0\cdots\Gamma^p\frac{1+(-1)^pi\eta\Gamma_{11}}{1+i\eta}\right)\\ &=-i\eta(-1)^{p+\frac{1}{2}p(p+1)}\,\mathcal{M}(-\eta)\,.\end{aligned}$$

We then find

$$\Omega|D(r,s)_\psi,\eta\rangle^{(0)}_{\rm R} = -i\eta(-1)^{\frac{1}{2}p(p+1)}\,|D(r,s)_\psi,-\eta\rangle^{(0)}_{\rm R}\,. \tag{2.17}$$

In the oscillator part, Ω exchanges tilded oscillators with untilded ones, yielding an overall change of η to $-\eta$. Using this fact and the result (2.15) we finally find:

$$\Omega|D(r,s)_\psi,\eta\rangle_{\rm R}|D(r,s)_{sgh},\eta\rangle_{\rm R} = -(-1)^{\frac{1}{2}p(p+1)}\,|D(r,s)_\psi,-\eta\rangle_{\rm R}|D(r,s)_{sgh},-\eta\rangle_{\rm R}\,, \tag{2.18}$$

so that the GSO-invariant R-R boundary state

$$|D(r,s)\rangle_{\rm R,U} = \frac{1}{2}\left(|D(r,s),+\rangle_{\rm R,U} + |D(r,s),-\rangle_{\rm R,U}\right)\,, \tag{2.19}$$

is Ω-invariant for $p = 1,5,9$, as expected for the physical BPS D-branes in the type I theory in the critical dimension.

Likewise, we can determine the Ω-projection on the D-branes by using the form of the quantum R-R string state in the $(-1/2,-3/2)$ picture[7], given in terms of the R-R gauge potential:

$$\begin{aligned} |C^{(p+1)}\rangle \;=\; & \frac{1}{2(p+1)!\sqrt{2}}\, C^{(p+1)}_{\mu_1\cdots\mu_{p+1}}\Big(\left(\mathcal{C}_{(10)}\Gamma^{\mu_1\cdots\mu_{p+1}}\Pi_+\right)_{AB}\,\cos\gamma_0\tilde{\beta}_0 \\ & + \left(\mathcal{C}_{(10)}\Gamma^{\mu_1\cdots\mu_{p+1}}\Pi_-\right)_{AB}\,\sin\gamma_0\tilde{\beta}_0\Big)\;|A\rangle_{-1/2}\otimes\widetilde{|B\rangle}_{-3/2}\,. \end{aligned} \tag{2.20}$$

We refer again to Appendix A for the details of this derivation. We state here the result:

$$\Omega\,|C^{(p+1)}\rangle = -(-1)^{\frac{1}{2}p(p+1)}\,|C^{(p+1)}\rangle\,. \tag{2.21}$$

Thus it is invariant for $p = 1,5,9$; the same values of p for which the R-R boundary states are Ω-invariant.

2.3 The Twisted R-R Sector

The orbifold of type IIB on $T^4/\mathcal{I}_4$ has 16 6-dimensional fixed planes with $(2,0)$ supersymmetry each. The R-R twisted ground state consists of two spinors of $SO(1,5)$ of the same chirality, either $\mathbf{4}$ or $\mathbf{4}'$. Therefore, the twisted R-R sector can contain the following field strengths:

$$\mathbf{4}\otimes\mathbf{4} = [1]+[3]_+\,,\qquad \mathbf{4}'\otimes\mathbf{4}' = [1]+[3]_-\,. \tag{2.22}$$

Depending on which chirality we choose for the spinors, the 3-form field strength will be self-dual or anti self-dual. Accordingly, we have twisted R-R boundary states $|D(r,s)\rangle_{\rm R,T}$ for $r = -1,1,3$. As we will see below, only those states with $r = -1,3$

[7]This particular approach presents the advantage over the usual one in the $(-1/2,-1/2)$ picture of making possible the description of spacetime-filling D-branes.

will survive the Ω projection, that is, only the twisted R-R scalar potential in (2.22) survives [18, 19]. However, note that (2.22) does not directly show the existence of a twisted 6-form potential, which was assumed in [19] and would be responsible for the twisted R-R tadpoles. This potential would be needed as well in order to account for an $r = 5$ twisted R-R boundary state that one can construct, as we show below. We see by direct construction of the corresponding quantum state that such a R-R potential does exist but it does not survive the Ω projection[8].

Consider first the action of Ω on the boundary states. The twisted R-R boundary state is given by

$$|D(r,s),\eta\rangle_{\mathrm{R,T}} = \frac{\tilde{Q}_{(r,s)}}{2}\,|D(r,s)_X\rangle_{\mathrm{T}}|D(r,s)_\psi,\eta\rangle_{\mathrm{R,T}}|D(r,s)_{gh}\rangle|D(r,s)_{sgh},\eta\rangle_{\mathrm{R}}\,, \quad (2.23)$$

which is constructed upon the twisted R-R ground state in the asymmetric $(-1/2,-3/2)$ picture [24]. This sector contains zero-modes:

$$|D(r,s),\eta\rangle^{(0)}_{\mathrm{R,T}} = \mathrm{e}^{i\eta\gamma_0\tilde{\beta}_0}M_{ab}\,|a\rangle_{-1/2,\mathrm{T}}\otimes|\widetilde{b}\rangle_{-3/2,\mathrm{T}}\,, \quad (2.24)$$

with

$$M_{ab}(\eta) = \left(\mathcal{C}_{(6)}\gamma^0\cdots\gamma^r\frac{1+i\eta\gamma}{1+i\eta}\right)_{ab}\,, \quad (2.25)$$

where a,b are spinor indices of $SO(1,6)$ and $\mathcal{C}_{(6)}$ is the charge conjugation matrix associated to the $SO(1,5)$ γ-matrices.

The action of Ω on this ground state results as before an exchange of pictures between left and right, hence we could proceed as before. Since the superghost part is the same as in the untwisted sector, the action of Ω on the $(-1/2,-3/2)$ picture is the same as given before in (2.15), and the effect on the zero-mode matrix M_{ab} is again a transposition:

$$\begin{aligned} M(\eta)^T &= -(-1)^{r+\frac{1}{2}r(r+1)}\left(\mathcal{C}_{(6)}\gamma^0\cdots\gamma^r\frac{1+(-1)^r i\eta\gamma}{1+i\eta}\right) \\ &= -i\eta(-1)^{\frac{1}{2}r(r+1)}\,M(-\eta)\,, \end{aligned}$$

where we have used the fact that fractional and truncated branes in type IIB on $T^4/\mathcal{I}_4$ only exist for r odd. Finally, using the above results and the fact that the twisted spinor indices anti-commute [26, 27], we find:

$$\Omega|D(r,s),\eta\rangle^{(0)}_{\mathrm{R,T}} = (-1)^{\frac{1}{2}r(r+1)}\,|D(r,s),-\eta\rangle^{(0)}_{\mathrm{R,T}}\,, \quad (2.26)$$

hence the GSO-invariant twisted R-R boundary state

$$|D(r,s)\rangle_{\mathrm{R,T}} = \frac{1}{2}\left(|D(r,s),+\rangle_{\mathrm{R,T}} + |D(r,s),-\rangle_{\mathrm{R,T}}\right)\,, \quad (2.27)$$

[8] On the other hand, one could choose the projection $\Omega = -1$ in the twisted sector, defined as the $\Omega\mathcal{J}$ projection in [26]. In that case the twisted R-R 6-form potential would survive in the orientifold.

is Ω invariant for $r = -1, 3$. For fractional branes s is even and for truncated branes s is odd [20]. Thus we find that there are non-BPS (truncated) branes $|D(-1,s)\rangle$ and $|D(3,s)\rangle$ branes, for s odd, in type I on $T^4/\mathcal{I}_4$. For the fractional (BPS) branes we must still find out which NS-NS twisted boundary states survive the projection. This is done in the next subsection.

We can find the same result using the quantum states of the R-R twisted sector in the $(-1/2,-3/2)$ picture:

$$\begin{aligned} |C^{(r+1)}\rangle_{\rm T} \;&=\; \frac{1}{\sqrt{2}(p+1)!} C^{(r+1)}_{\mu_1\ldots\mu_{r+1}} \left(\left(\mathcal{C}_{(6)}\gamma^{\mu_1\ldots\mu_{p+1}}\Pi_+\right)_{ab} \cos\gamma_0\tilde{\beta}_0\right. \\ &\quad + \left.\left(\mathcal{C}_{(6)}\gamma^{\mu_1\ldots\mu_{p+1}}\Pi_-\right)_{ab} \sin\gamma_0\tilde{\beta}_0\right) |a\rangle_{-1/2,{\rm T}} \otimes |\widetilde{b}\rangle_{-3/2,{\rm T}}\,. \end{aligned} \tag{2.28}$$

Using (A.20) from Appendix A we find

$$\Omega\,|C^{(r+1)}\rangle_{\rm T} = (-1)^{\frac{1}{2}r(r+1)}\,|C^{(r+1)}\rangle_{\rm T}\,, \tag{2.29}$$

hence the twisted scalar and its dual, the 4 form potential, survive the Ω projection, whereas the 6-form potential does not survive, which is in agreement with the results of [17, 18, 19].

2.4 The Twisted NS-NS Sector

The boundary state in the twisted NS-NS sector

$$|D(r,s),\eta\rangle_{\rm NS,T} = \frac{\tilde{T}_{(r,s)}}{2}\,|D(r,s)_X\rangle_{\rm T}|D(r,s)_\psi,\eta\rangle_{\rm NS,T}|D(r,s)_{gh}\rangle|D(r,s)_{sgh},\eta\rangle_{\rm NS} \tag{2.30}$$

is constructed upon the NS-NS ground state in the $(-1,-1)$ superghost picture. This boundary state contains a sector for the fermion zero-modes coming from the compact directions 6, 7, 8 and 9 :

$$|D(r,s),\eta\rangle^{(0)}_{\rm NS,T} = m_{\alpha\beta}(\eta)\,|\alpha\rangle_{-1,{\rm T}} \otimes |\widetilde{\beta}\rangle_{-1,{\rm T}}\,, \tag{2.31}$$

where α,β are spinor indices of $SO(4)$. Furthermore, from the overlapping condition for the zero modes, we find:

$$m_{\alpha\beta}(\eta) = \left(\mathcal{C}_{(4)}\Pi_{(s)}\frac{1-i\eta\bar{\gamma}}{1-i\eta}\right)_{\alpha\beta}\,, \tag{2.32}$$

where $\Pi_{(s)}$ is the product of s gamma-matrices:

$$\begin{aligned} \Pi_{(s)} &= \bar{\gamma}^6\bar{\gamma}^7\cdot\ldots\,, \qquad s\neq 0\,, \\ \Pi_{(s)} &= 1\,, \qquad s=0\,, \end{aligned} \tag{2.33}$$

and $\bar{\gamma} = -\bar{\gamma}^6\bar{\gamma}^7\bar{\gamma}^8\bar{\gamma}^9$ is the chirality matrix in four Euclidean dimensions. Moreover, $\mathcal{C}_{(4)}$ represents the charge-conjugation matrix in the present context. Our conventions for the $SO(4)$ gamma-matrices and the action of the fermionic zero-modes on the ground state can be found in Appendix B.

The ground state, which is constructed with anti-commuting spin fields of the same chirality, transforms as follows under Ω:

$$\Omega\left(|\alpha\rangle_{-1,\mathrm{T}}\otimes\widetilde{|\beta\rangle}_{-1,\mathrm{T}}\right)=(\bar{\gamma})^{\alpha}{}_{\delta}\,|\beta\rangle_{-1,\mathrm{T}}\otimes\widetilde{|\delta\rangle}_{-1,\mathrm{T}}=(\bar{\gamma})^{\beta}{}_{\delta}\,|\delta\rangle_{-1,\mathrm{T}}\otimes\widetilde{|\alpha\rangle}_{-1,\mathrm{T}}\,. \tag{2.34}$$

A proof of this result is presented in Appendix B. Using this result we obtain the following action of Ω on the boundary state corresponding to the vacuum sector:

$$\begin{aligned}\Omega\,|D(r,s),\eta\rangle^{(0)}_{\mathrm{NS,T}} &= \left(m(\eta)^{T}\,\bar{\gamma}\right)_{\alpha\beta}|\alpha\rangle_{-1,\mathrm{T}}\otimes\widetilde{|\beta\rangle}_{-1,\mathrm{T}}\\ &= -(-1)^{\frac{1}{2}s(s-1)}\,|D(r,s),-\eta\rangle^{(0)}_{\mathrm{NS,T}}\,.\end{aligned} \tag{2.35}$$

with s even. Note that the twisted NS-NS boundary state is $\mathcal{I}_4$-invariant for $s = even$ [20]. On the other hand, in the sector bilinear in the oscillators Ω acts again by changing η by $-\eta$. Thus we finally deduce:

$$\Omega\,|D(r,s),\eta\rangle_{\mathrm{NS,T}}=-(-1)^{\frac{1}{2}s(s-1)}\,|D(r,s),-\eta\rangle_{\mathrm{NS,T}}\,, \tag{2.36}$$

hence the GSO-projected boundary state

$$|D(r,s)\rangle\rangle_{\mathrm{NS,T}}=\frac{1}{2}\left(|D(r,s),+\rangle_{\mathrm{NS,T}}+|D(r,s),-\rangle_{\mathrm{NS,T}}\right)\,,\qquad s\,=\,even \tag{2.37}$$

is Ω-invariant only for $s = 2$. Accordingly, only branes with $s = 2$ in type I on $T^4/\mathcal{I}_4$ carry a NS-NS twisted sector.

3 Non-BPS Branes in Type I on $T^4/\mathbb{Z}_2$

Before describing in detail the stability of the non-BPS branes in the type I orbifold, we summarise the brane spectrum that we have found with the analysis based on the boundary states. Recall that the D-branes are denoted by $|D(r,s)\rangle$, with r the number of Neumann directions along the orbifold fixed plane and s the number of Neumann directions along T^4. From the point of view of the closed string the orientifold is described in terms of crosscaps. Since there are orientifold 9- and 5- planes, we must consider a 9-crosscap $|C9\rangle$ and 5-crosscaps $|C5\rangle$. The type I orbifold can be considered as an orientifold of type IIB on $T^4/\mathcal{I}_4$. Accordingly the branes in the type I orbifold will be modified by the addition of the crosscaps:

$$|D\rangle\longrightarrow|D\rangle_{T^4/\mathbb{Z}_2}=\frac{1}{\sqrt{2}}\left(|D\rangle+|C9\rangle+|C5\rangle\right)\,, \tag{3.1}$$

where we would introduce one 5-crosscap for each fixed plane, and we have chosen this particular normalisation to account for the normalisation of the orientifold projectors in the open string channel.

Boundary State	Ω-invariance	$\mathcal{I}_4$-invariance
NS-NS untwisted	$\forall\ r, s$	$\forall\ r, s$
R-R untwisted	$r+s=1,5,9$	$s=$ even
R-R twisted	$r=-1,3$	$\forall\ r, s$
NS-NS twisted	$s=2$	$s=$ even

Table 1: **Ω and $\mathcal{I}_4$ -invariance of the boundary states.** This table shows for which values of r and s each boundary state is invariant under the operations Ω and $\mathcal{I}_4$.

In type I on $T^4/\mathbb{Z}_2$ we find also BPS fractional branes, which have states in all sectors twisted and untwisted, and include the crosscaps as well:

$$|D(r,s)\rangle_{f,T^4/\mathbb{Z}_2} = \frac{1}{\sqrt{2}}\left(|D(r,s)\rangle_f + |C9\rangle + |C5\rangle\right) . \tag{3.2}$$

The values of r and s for which they exist are such that the boundary state in every sector is invariant under both Ω and $\mathcal{I}_4$. From the results of the previous section, shown in table 1, we find these values to be $r=-1,3$ and $s=2$, i.e. the orbifold theory has an *instantonic* D1 and a D5 as fractional branes.

BPS Bulk branes can be described as branes with only untwisted sectors. A possibility is to have r and s such that the untwisted sectors are invariant, but not the twisted sectors. This yields $r=1,5$ and $s=0,4$, which accounts for $D1$, $D5$ and $D9$ branes. On the other hand, two fractional branes with opposite twisted R-R charge can join and produce a bulk brane that can move off the fixed planes. Therefore we also can have bulk branes for the same values of r and s as for fractional branes. Bulk branes can be represented as follows:

$$|D(r,s)\rangle_{b,T^4/\mathbb{Z}_2} = \frac{1}{\sqrt{2}}\left(|D(r,s)\rangle_b + |C9\rangle + |C5\rangle\right) . \tag{3.3}$$

In the type I orbifold, only those truncated branes whose twisted R-R state survives the orbifold symmetry are Z-charged non-BPS branes. Moreover, some non-BPS branes of type I have values r and s such that the twisted R-R charge in the type I orbifold exist. Thus these branes are also truncated in the orbifold. We find that $\mathbb{Z}$-charged non-BPS branes can exist for $r=-1,3$ and $s=0,1,3,4$, since for these values $|D(r,s)\rangle_{\mathrm{R,T}}$ is Ω-invariant but $|D(r,s)\rangle_{\mathrm{R}}$ is not. These branes can be represented by:

$$|D(r,s)\rangle_{t,T^4/\mathbb{Z}_2} = \frac{1}{\sqrt{2}}\left(|D(r,s)\rangle_t + |C9\rangle + |C5\rangle\right) . \tag{3.4}$$

These branes include the non-BPS D-instanton and D7 brane of type I, which have twisted R-R charge in the type I orbifold. This $\mathbb{Z}$-charged D7 brane appears only for the orientation $(3,4)$.

BPS branes	
Fractional & Bulk branes	$r=-1,3$ $s=2$
Bulk branes	$r=1,5$ $s=0,4$

Table 2: **BPS branes in type I on $T^4/\mathbb{Z}_2$.** This table shows for which (r,s) we can have either BPS fractional or bulk branes in the type I orbifold.

Non-BPS branes		
$\mathbb{Z}$-charge	$r=-1,3$ $s=0,1,3,4$	
$\mathbb{Z}_2$-charge	$r=1,5$ $s=1,2,3$	$r=$ even $\forall\, s$

Table 3: **Non-BPS branes in type I on $T^4/\mathbb{Z}_2$.** Non-BPS branes can have $\mathbb{Z}$-charge, if they have a twisted R-R sector, or $\mathbb{Z}_2$ charge if they do not have any R-R sector at all. The table shows for which (r,s) the boundary states exist in the type I orbifold.

The last possibility consists of those branes which have neither untwisted nor twisted R-R charge. These include those branes which are BPS in the type IIB orbifold, but become non-BPS in the orientifold, and also those branes in type I which do not obtain any R-R charge after the orbifolding. They are described solely by an untwisted NS-NS sector, and in the type I orbifold can be represented by:

$$|D(r,s)\rangle_{t,T^4/\mathbb{Z}_2} = \frac{1}{\sqrt{2}}\left(|D(r,s)\rangle_{\mathrm{NS,U}} + |C9\rangle + |C5\rangle\right)\ . \tag{3.5}$$

The values of r and s for which they can exist is deduced by imposing that no R-R sector must survive. The two possibilities are either $r=1,5$ and $s=1,2,3$, or $r=$ *even* and $s=0,...,4$. These branes include the non-BPS D0, D7 and D8 branes of type I. The D7 and D8 branes appear with different orientations, namely, $(5,2)$, $(4,3)$, $(5,3)$, $(4,4)$. As it happens with these branes in type I [13, 14], the truncated branes of the type I orbifold may be stabilised by the action of the orientifold. How exactly this takes place will be described in the next section.

4 Stability of the Non-BPS Branes

In this section we study the stability of the non-BPS branes presented in the previous section. We study the $\mathbb{Z}_2$ and the $\mathbb{Z}$ branes independently.

It is crucial to notice that the tadpole-cancelling D9 and D5 branes in type I on $T^4/\mathcal{I}_4$ are bulk branes[9]. Therefore the 9- and 5-crosscaps of the type I orbifold have boundary states in the untwisted sectors only[10]:

$$|C9\rangle = |C9\rangle_{\rm NS,U} + |C9\rangle_{\rm R,U}\,, \qquad |C5\rangle = |C5\rangle_{\rm NS,U} + |C5\rangle_{\rm R,U}\,. \tag{4.1}$$

The detailed form of these crosscaps is given in Appendix C.

4.1 Stable Non-BPS Branes with $\mathbb{Z}_2$ Charge

Our analysis of the stability of these branes follows closely the procedure used in [14]. We introduce a parameter m_p to renormalise the tension of these branes, which will be fixed by imposing that there are no tachyons in the open string spectrum on that brane. In order to achieve this, we calculate the interaction between two of these non-BPS branes and translate the result to open string channel. Then, we impose that there are no tachyons in the open string spectrum, which will fix the parameter m_p. Only positive values of m_p will be considered to produce a consistent stable brane.

In the orientifold, the $\mathbb{Z}_2$-charged branes are represented as follows:

$$|D(r,s)\rangle_{T^4/\mathbb{Z}_2} = \frac{1}{\sqrt{2}}\left(|D(r,s)\rangle_{\rm NS,U} + |C9\rangle + |C5\rangle\right)\,. \tag{4.2}$$

The relevant open string amplitudes are given by the annulus and Möbius amplitudes, which can be obtained from the closed channel as follows:

$$\begin{aligned}
\mathcal{A} &= \frac{1}{2}{}_{\rm NS,U}\langle D(r,s)|\mathcal{D}|D(r,s)\rangle_{\rm NS,U}\,, \qquad (4.3)\\
\mathcal{M}_9 + \mathcal{M}_5 &= \frac{1}{2}{}_{\rm NS,U}\langle D(r,s)|\mathcal{D}|C9\rangle + \frac{1}{2}{}_{\rm NS}\langle D(r,s)|\mathcal{D}|C5\rangle\,,
\end{aligned}$$

where $\mathcal{D}$ is the closed string propagator

$$\mathcal{D} = \frac{\alpha'}{4\pi}\int_{|z|\le 1}\frac{d^2z}{|z|^2}\, z^{L_0-a}\,\bar{z}^{\tilde{L}_0-a}\,, \tag{4.4}$$

[9]This can be deduced from the fact that the twisted R-R state associated with the 6-form potential is odd under Ω, which agrees with the fact that the tadpole associated with the twisted 6-form is identically zero [17, 18, 19]. The normalisation of the boundary states in the orbifold involves the trace of the representation matrices of the orientifold group on the open string sector. This trace is exactly zero for the case of the twisted R-R 6-form, hence the D9 and D5 branes do not carry a twisted R-R sector, and by supersymmetry they do not carry any twisted NS-NS sector either.

[10]Consistency conditions of type I on the orbifold $T^4/\mathcal{I}_4$ using the boundary state formalism was first considered in [16].

where $a = 1/2$ for the untwisted NS-NS sector and $a = 0$ in the R-R sector (twisted and untwisted) and the twisted NS-NS sector. Since the crosscaps are *bulk* as well, they will interact with the $\mathbb{Z}_2$-charged branes only through the untwisted NS-NS sector.

Note that the normalisation of the brane is modified by the fact that the orbifold is compact, namely:

$$|D(r,s),\eta\rangle_{\rm NS,U} = m_p N_p\, |D(r,s)_X\rangle |D(r,s)_\psi,\eta\rangle_{\rm NS,U} |D(r,s)_{gh}\rangle |D(r,s)_{sgh}\rangle_{\rm NS}\,, \qquad (4.5)$$

with

$$N_p = \frac{T_p}{2}\left(\frac{2\pi R}{\Phi}\right)^2 (2\pi R)^{-4+s}\,, \qquad (4.6)$$

where T_p is the tension of a BPS Dp-brane in type II:

$$T_p = \sqrt{\pi}(2\pi\sqrt{\alpha'})^{3-p}\,, \qquad p = r+s\,. \qquad (4.7)$$

Moreover, the bosonic sector is modified as follows:

$$\begin{aligned} |D(r,s)_X\rangle &= \delta^{(5-p)}(q^i - y^i)\left(\sum_{m\in\mathbb{Z}} \mathrm{e}^{iq_m \frac{mR}{\alpha'}}\right)^s \left(\sum_{n\in\mathbb{Z}} \mathrm{e}^{iq_n \frac{n}{R}}\right)^{4-s} \times \\ &\times \prod_{n=1}^{\infty} \mathrm{e}^{-\frac{1}{n}\alpha_{-n}\cdot\mathcal{S}\cdot\tilde{\alpha}_{-n}}\,|k,m,n=0\rangle\,. \end{aligned} \qquad (4.8)$$

With this information and using the explicit form of the crosscaps given in Appendix C, we can compute the required amplitudes. After standard operations we find in the closed string channel

$$\begin{aligned} {}_{\rm NS,U}\langle D(r,s)|\mathcal{D}|D(r,s)\rangle_{\rm NS,U} &= \frac{\alpha'\pi}{2}(m_p N_p)^2 V_{r+1}\Phi^4 (2\pi^2\alpha')^{-(\frac{5-r}{2})} \int_0^\infty d\ell\, \ell^{-(\frac{5-r}{2})} \times \\ \times \left(\sum_{n\in\mathbb{Z}} \mathrm{e}^{-\pi\ell\frac{\alpha'}{2}(\frac{n}{R})^2}\right)^{4-s} & \left(\sum_{m\in\mathbb{Z}} \mathrm{e}^{-\pi\ell\frac{(mR)^2}{2\alpha'}}\right)^s \frac{1}{2}\left\{\left(\frac{f_3(q)}{f_1(q)}\right)^8 - \left(\frac{f_4(q)}{f_1(q)}\right)^8\right\}\,, \end{aligned} \qquad (4.9)$$

with $q = \mathrm{e}^{-\pi\ell}$. We have introduced the self-dual volume Φ defined as[11]

$$\langle n,m|n'm'\rangle = \Phi\,\delta_{nn'}\delta_{mm'}\,, \qquad (4.10)$$

for each compact direction. The closed string amplitudes related to the Möbius strips are given by:

$$\begin{aligned} {}_{\rm NS,U}\langle D(r,s)|\mathcal{D}|C9\rangle_{\rm NS,U} &= -\frac{\alpha'\pi}{2}(m_p N_p)\mathcal{N}_9\Phi^4 V_{r+1} 2^{\frac{9-p}{2}} \int_0^\infty d\ell \left(\sum_{m\in\mathbb{Z}} \mathrm{e}^{-\pi\ell\frac{(mR)^2}{2\alpha'}}\right)^s \times \\ \times \frac{1}{2}\left\{\left(\frac{f_3(iq)}{f_1(iq)}\right)^{p-1}\right. & \left.\left(\frac{f_4(iq)}{f_2(iq)}\right)^{9-p} - \left(\frac{f_4(iq)}{f_1(iq)}\right)^{p-1}\left(\frac{f_3(iq)}{f_2(iq)}\right)^{9-p}\right\}\,, \end{aligned} \qquad (4.11)$$

[11] See [23] for more details.

and

$$
\begin{aligned}
{}_{\rm NS,U}\langle D(r,s)|\mathcal{D}|C5\rangle_{\rm NS,U} &= -\frac{\alpha'\pi}{2}(m_pN_p)\mathcal{N}_5\Phi^4V_{r+1}2^{\frac{5-r+s}{2}}\int_0^\infty d\ell\left(\sum_{n\in\mathbb{Z}}\mathrm{e}^{-\pi\ell\frac{\alpha'}{2}(\frac{n}{R})^2}\right)^{4-s}\times\\
&\times\frac{1}{2}\left\{\left(\frac{f_3(iq)}{f_1(iq)}\right)^{3+r-s}\left(\frac{f_4(iq)}{f_2(iq)}\right)^{5-r+s}-\left(\frac{f_4(iq)}{f_1(iq)}\right)^{3+r-s}\left(\frac{f_3(iq)}{f_2(iq)}\right)^{5-r+s}\right\}, \qquad (4.12)
\end{aligned}
$$

where $\mathcal{N}_9$ and $\mathcal{N}_5$ are the normalisation factors of the $C9$ and $C5$ given in Appendix C. In order to obtain the amplitudes in open string channel we perform a modular transformation, $\ell = t^{-1}$, for the cylinder and $\ell = (4t)^{-1}$ for the Möbius strip. After using the modular properties of the functions[12] f_i, we find:

$$
\begin{aligned}
\mathcal{A} &= \frac{\alpha'\pi}{2}(m_pN_p)^2V_{r+1}\Phi^4(2\pi^2\alpha')^{-(\frac{5-r}{2})}R^{4-2s}(\alpha')^{-2+s}\int_0^\infty\frac{dt}{t}t^{-(\frac{r+1}{2})}\times \qquad (4.13)\\
&\times\left(\sum_{m\in\mathbb{Z}}\mathrm{e}^{-2\pi t\frac{(mR)^2}{\alpha'}}\right)^{4-s}\left(\sum_{n\in\mathbb{Z}}\mathrm{e}^{-2\pi t\alpha'(\frac{n}{R})^2}\right)^{s}\left\{\left(\frac{f_3(\tilde{q})}{f_1(\tilde{q})}\right)^8-\left(\frac{f_2(\tilde{q})}{f_1(\tilde{q})}\right)^8\right\},
\end{aligned}
$$

with $\tilde{q} = \mathrm{e}^{-\pi t}$.

$$
\begin{aligned}
\mathcal{M}_9 &= -\frac{\pi}{2}(m_pN_p)\mathcal{N}_9\Phi^4V_{r+1}R^{-s}(\alpha')^{1+\frac{s}{2}}2^{-p+1+\frac{3}{2}s}\int_0^\infty\frac{dt}{t}t^{-\frac{r+1}{2}}\left(\sum_{n\in\mathbb{Z}}\mathrm{e}^{-8\pi t\alpha'(\frac{n}{R})^2}\right)^{s}\times\\
&\times\left\{\mathrm{e}^{i\frac{\pi}{4}(p-5)}\left(\frac{f_4(i\tilde{q})}{f_1(i\tilde{q})}\right)^{p-1}\left(\frac{f_3(i\tilde{q})}{f_2(i\tilde{q})}\right)^{9-p}\right. \qquad (4.14)\\
&\qquad\left.-\ \mathrm{e}^{-i\frac{\pi}{4}(p-5)}\left(\frac{f_3(i\tilde{q})}{f_1(i\tilde{q})}\right)^{p-1}\left(\frac{f_4(i\tilde{q})}{f_2(i\tilde{q})}\right)^{9-p}\right\},
\end{aligned}
$$

and

$$
\begin{aligned}
\mathcal{M}_5 &= -\frac{\pi}{2}(m_pN_p)\mathcal{N}_5\Phi^4V_{r+1}R^{4-s}(\alpha')^{-1+\frac{s}{2}}2^{3-r-\frac{s}{2}}\int_0^\infty\frac{dt}{t}t^{-(\frac{r+1}{2})}\left(\sum_{m\in\mathbb{Z}}\mathrm{e}^{-8\pi t\frac{(mR)^2}{\alpha'}}\right)^{4-s}\times\\
&\times\left\{\mathrm{e}^{-i\frac{\pi}{4}(1-r+s)}\left(\frac{f_4(i\tilde{q})}{f_1(i\tilde{q})}\right)^{3+r-s}\left(\frac{f_3(i\tilde{q})}{f_2(i\tilde{q})}\right)^{5-r+s}\right. \qquad (4.15)\\
&\qquad\left.-\ \mathrm{e}^{i\frac{\pi}{4}(1-r+s)}\left(\frac{f_3(i\tilde{q})}{f_1(i\tilde{q})}\right)^{3+r-s}\left(\frac{f_4(i\tilde{q})}{f_2(i\tilde{q})}\right)^{5-r+s}\right\}.
\end{aligned}
$$

The open string spectrum for these non-BPS branes in the type I orbifold is given by the total amplitude $\mathcal{A}_{total} = \mathcal{A}+\mathcal{M}_9+\mathcal{M}_9^*+\mathcal{M}_5+\mathcal{M}_5^*$. Expanding in powers

[12] The explicit form of the modular properties of these functions with imaginary argument can be found in [14].

of $\tilde{q}$, for $t \to \infty$, we can analyse the tachyonic states. Using the explicit form of the normalisation factors, we obtain the following leading term in the amplitude $\mathcal{A}_{total}$:

$$\mathcal{A}_{total} \simeq m_p (8\pi^2\alpha')^{-(\frac{r+1}{2})} \int \frac{dt}{t} t^{-(\frac{r+1}{2})} \tilde{q}^{-1} \times \\ \times \left\{ \frac{m_p}{\sqrt{2}} + 2^s \sin[\frac{\pi}{4}(p-5)] + 2^{4-s} \sin[\frac{\pi}{4}(1-r+s)] \right\} + \dots , \tag{4.16}$$

which corresponds to the contribution from the tachyon. In order not to have tachyons this contribution must therefore vanish:

$$m_p = -2^{s+\frac{1}{2}} \sin[\frac{\pi}{4}(p-5)] - 2^{4-s+\frac{1}{2}} \sin[\frac{\pi}{4}(1-r+s)] . \tag{4.17}$$

Given the value of r and s for the branes with only NS-NS untwisted part given in table 3, we find the following branes with m_p a positive integer:

$m_p = 6$	$m_p = 17$
$D(1,3)$	$D(0,4)$
$D(5,1)$	$D(2,0)$
	$D(4,0)$

We also find branes with $m_p > 0$ but not integer:

$m_p = 2\sqrt{2}$	$m_p = 4\sqrt{2}$	$m_p = 8\sqrt{2}$
$D(2,1)$	$D(1,2)$	$D(5,2)$
		$D(0,3)$
		$D(4,1)$

These branes will still produce an integer multiple of open strings from the cylinder amplitude. For the rest of the cases we find m_p either negative or zero. The cases with a remarkably large integer value for m_p suggests the possibility that the brane might be a composite of lighter branes. Notice that the non-BPS D0 and D8 branes of type I are not stable anymore in the type I orbifold. On the contrary, the non-BPS D7 brane $D(5,2)$ is stable.

Among these branes, only those which moreover do not have tachyons in the Neumann-Dirichlet strings stretching between the non-BPS brane and the tadpole-cancelling branes in the background are absolutely stable. Since in this case only NS-NS interactions are relevant, in order to do this analysis it is enough to consider the tachyon-masses in the NS-spectrum of the ND strings. For the strings stretching between the non-BPS brane and the D9 brane we have

$$M^2 = \sum_{s\,(NN)\,directions} \left(\frac{n_i}{R_i}\right)^2 + \frac{1}{\alpha'}\left(\frac{5-p}{8}\right) . \tag{4.18}$$

For $p = r + s \leq 5$ there are no tachyons, for any radius. Since the ground state of this sector ($n_i = 0$, $\forall i$) is not projected out of the spectrum, when $p > 5$ the brane contains always at least one tachyon. On the other hand, for the strings stretching between the non-BPS brane and the D5 brane we have

$$M^2 = \sum_{4-s\,(DD)\,directions} \left(\frac{m_i R_i}{\alpha'}\right)^2 + \frac{1}{\alpha'}\left(\frac{1-r+s}{8}\right) . \tag{4.19}$$

For $r - s \leq 1$ there are no tachyons, for any radius. Similarly as above, for $r - s > 1$ the brane contains at least one tachyon in this sector. Taking this analysis into account, using with the previous results, we find that only the D3 and D4 branes

$D(1,3)$	$D(0,4)$	$D(2,1)$	$D(1,2)$	$D(0,3)$

do not contain any tachyons at all in their spectrum. It is interesting to compare this result with type I theory with a symplectic crosscap, where the non-BPS D3 and D4 branes are stable [13, 14].

4.2 Stable Non-BPS Branes with $\mathbb{Z}$ Charge

These are the branes that carry a twisted R-R sector, namely

$$r = -1, 3\,, \qquad s = 0, 1, 3, 4\,, \tag{4.20}$$

and they have the following form:

$$|D(r,s)\rangle_{T^4/\mathbb{Z}_2} = \left(|D(r,s)\rangle_{\mathrm{NS,U}} + |D(r,s)\rangle_{\mathrm{R,T}} + |C5\rangle + |C9\rangle\right) . \tag{4.21}$$

As for the truncated branes in a type IIB orbifold, they may be stable for some range of the radii. As before, we can obtain the amplitude of open strings on one of these branes using the boundary states:

$$\begin{aligned} \mathcal{A}_{total} &= {}_{T^4/\mathbb{Z}_2}\langle D(r,s)|\mathcal{D}|D(r,s)\rangle_{T^4/\mathbb{Z}_2} \\ &= \mathcal{A}_{NS-NS} + \mathcal{A}_{R-R,T} + \mathcal{M}_{9,NS-NS} + \mathcal{M}^*_{9,NS-NS} \\ &\quad + \mathcal{M}_{5,NS-NS} + \mathcal{M}^*_{5,NS-NS}\,. \end{aligned} \tag{4.22}$$

The correspondence between the spin structures of the loop and tree channels is as follows:

$$\mathcal{A}_{NS-NS} = \int_0^\infty \frac{dt}{2t}\mathrm{Tr}_{NS,R}\left(\frac{1}{4}(-1)^{F_s}\mathrm{e}^{-2\pi t L_0}\right) \longleftrightarrow \frac{1}{2}{}_{\mathrm{NS,U}}\langle D(r,s)|\mathcal{D}|D(r,s)\rangle_{\mathrm{NS,U}}\,,$$

$$\mathcal{A}_{R-R,T} = \int_0^\infty \frac{dt}{2t}\mathrm{Tr}_{NS}\left(\frac{1}{4}(-1)^{F+G}\mathcal{I}_4\,\mathrm{e}^{-2\pi t L_0}\right) \longleftrightarrow \frac{1}{2}{}_{\mathrm{R,T}}\langle D(r,s)|\mathcal{D}|D(r,s)\rangle_{\mathrm{R,T}}\,,$$

$$\mathcal{M}_{9,NS-NS} = \int_0^\infty \frac{dt}{2t}\mathrm{Tr}_{NS}\left(\frac{1}{4}\Omega\,\mathrm{e}^{-2\pi t L_0}\right) \longleftrightarrow \frac{1}{2}{}_{\mathrm{NS,U}}\langle D(r,s)|\mathcal{D}|C9\rangle_{\mathrm{NS,U}}\,,$$

$$\mathcal{M}_{5,NS-NS} = \int_0^\infty \frac{dt}{2t}\mathrm{Tr}_{NS}\left(\frac{1}{4}(-1)^{F+G}\mathcal{I}_4\Omega\,\mathrm{e}^{-2\pi t L_0}\right) \longleftrightarrow \frac{1}{2}{}_{\mathrm{NS,U}}\langle D(r,s)|\mathcal{D}|C5\rangle_{\mathrm{NS,U}}\,.$$

where F_s is the space-time fermion number and $(-1)^{F+G}$ is the GSO projection operator including the super-ghosts. Furthermore, we take into account that

$$\mathrm{Tr}_R\left((-1)^{F+G}\mathcal{I}_4\,\mathrm{e}^{-2\pi tL_0}\right)=0\,, \tag{4.23}$$

due to the presence of zero modes in the R-vacuum, and

$$\mathrm{Tr}_R\left(\Omega\,\mathrm{e}^{-2\pi tL_0}\right)=0\,, \tag{4.24}$$

since Ω acts as a product of gamma-matrices on the R-vacuum, and finally

$$\mathrm{Tr}_R\left((-1)^{F+G}\mathcal{I}_4\Omega\,\mathrm{e}^{-2\pi tL_0}\right)=0\,, \tag{4.25}$$

since although the action of Ω and $\mathcal{I}_4$ compensate it still vanishes due to $(-1)^{F+G}$. Accordingly we can write the total amplitude in terms of an open string loop as follows:

$$\mathcal{A}_{total}=\int_0^\infty\frac{dt}{2t}\mathrm{Tr}\left\{(-1)^{F_s}\left(\frac{1+(-1)^{F+G}\mathcal{I}_4}{2}\right)\left(\frac{1+\Omega}{2}\right)\mathrm{e}^{-2\pi tL_0}\right\}\,. \tag{4.26}$$

The open strings on these D-branes have winding and Kaluza-Klein modes

$$M^2=\sum_{4-s(DD)directions}\left(\frac{mR}{\alpha'}\right)^2+\sum_{s(NN)directions}\left(\frac{n}{R}\right)^2+\frac{1}{\alpha'}\left(N-\frac{1}{2}\right)\,, \tag{4.27}$$

hence for a certain range of the radii there are no tachyons in the spectrum: $R\geq\sqrt{\alpha'/2}$ for the DD-directions and $R\leq\sqrt{2\alpha'}$ for the NN-directions. Moreover, at particular radii, the tachyon modes become massless. In the case of non-BPS branes in type II orbifolds, these critical radii coincide with the critical radii where the brane has a vanishing 1-loop amplitude [28]. In this case, however, this is not possible since there is a remaining NS-NS interaction of the brane with the crosscaps that cannot be cancelled.

On the other hand, for the analysis of tachyons in the strings stretched from the non-BPS brane to the tadpole-cancelling branes we can use the analysis carried out before for the Z_2-charged non-BPS branes. Using (4.18) and (4.19) one can see that the *instantonic* branes $|D(-1,0)\rangle_{T^4/Z_2}$, $|D(-1,1)\rangle_{T^4/Z_2}$, $|D(-1,3)\rangle_{T^4/Z_2}$ and $|D(-1,4)\rangle_{T^4/Z_2}$ have no tachyons in this sector at any radius.

On the other hand, the branes $|D(3,0)\rangle_{T^4/Z_2}$, $|D(3,1)\rangle_{T^4/Z_2}$, $|D(3,3)\rangle_{T^4/Z_2}$ and $|D(-3,4)\rangle_{T^4/Z_2}$ contain at least a tachyon in this sector. Accordingly, the *instantonic* truncated branes listed above are fully stable if the radii take the values $R\geq\sqrt{\alpha'/2}$ for the DD-directions and $R\leq\sqrt{2\alpha'}$ for the NN-directions, for each particular brane. Finally, we observe that the D-instanton of type I, which becomes a truncated brane with twisted R-R charge in the type I orbifold, can be stable in such theory.

5 Conclusions

In this article we have presented a thorough analysis of the action of the Ω projection on the boundary states of type IIB theory in a $T^4/\mathcal{I}_4$ orbifold. Of particular interest are the R-R sectors and the twisted NS-NS sector. We have shown how to derive the action of Ω in the R-R sectors of the covariant boundary states, which are formulated in the asymmetric picture $(-1/2, -3/2)$, for which the superghost zero-modes play an important role. In the twisted NS-NS sector, where there are no superghost zero modes, we have implemented the action of Ω through the algebra of fermionic zero modes. Since these results rely on basic features of the covariant boundary states, which are common to other theories, our approach could be extended to other orbifold and orientifold theories in order to derive their complete D-brane spectrum in a systematic way.

Using the results of our analysis of the Ω projection, and taking into account the action of the orbifold on the different sectors, we have derived which boundary states are present in the type I orbifold. From this we have derived the spectrum of BPS and non-BPS D-branes of type I on $T^4/\mathcal{I}_4$. Regarding the non-BPS D-branes, they are divided into $\mathbb{Z}$-charged branes, which have a twisted R-R sector, and $\mathbb{Z}_2$-charged branes, which are described by a untwisted NS-NS sector only.

The analysis of the stability of these non-BPS D-branes has been carried out by looking at the open string tachyons on the branes. For the $\mathbb{Z}_2$-charged non-BPS branes we have found a number of them which have no tachyons in their spectrum, namely: $|D(1,3)\rangle$, $|D(5,1)\rangle$, $|D(0,4)\rangle$, $|D(2,0)\rangle$, $|D(4,0)\rangle$, $|D(2,1)\rangle$, $|D(1,2)\rangle$, $|D(5,2)\rangle$, $|D(0,3)\rangle$, $|D(4,1)\rangle$. Among these branes, some of them can have tachyons in the spectrum of the open strings stretching to the tadpole cancelling branes. In fact, we have found only five of the above branes which present no tachyons in this sector and therefore can be considered as fully stable branes. These branes are D3 and D4 branes: $|D(1,3)\rangle$, $|D(0,4)\rangle$, $|D(2,1)\rangle$, $|D(1,2)\rangle$ and $|D(0,3)\rangle$.

On the other hand, the non-BPS $\mathbb{Z}$-charged branes may have no tachyons on their brane spectrum for a particular range of the radii, namely, $R \geq \sqrt{\alpha'/2}$ for the DD-directions and $R \leq \sqrt{2\alpha'}$ for the NN-directions, as occurs in the type IIB orbifold. Moreover, the truncated instantonic branes: $|D(-1,0)\rangle$, $|D(-1,1)\rangle$, $|D(-1,3)\rangle$, $|D(-1,4)\rangle$, do not have any tachyons in the open strings stretching to the tadpole cancelling branes, so the can be considered as fully stable for the range of the radii mentioned above. The other branes, namely, $|D(3,0)\rangle$, $|D(3,1)\rangle$, $|D(3,3)\rangle$, $|D(3,4)\rangle$, have tachyons in the open strings stretching to the tadpole cancelling branes.

It is interesting to analyse the fate of the well-known non-BPS branes of type I theory after the orbifolding. The D0 and D8 branes become non-BPS $\mathbb{Z}_2$-charged branes. However, none of them are stable. The D7 branes becomes either a $\mathbb{Z}$-charged branes, for the orientation $(3,4)$, or a $\mathbb{Z}_2$-charged brane, for the orientations $(5,2)$ and $(4,3)$. The $D(3,4)$ is in principle stable for radii $R \leq \sqrt{2\alpha'}$, however it contains tachyons in the open strings stretching to the tadpole-cancelling branes. On the other hand, the $\mathbb{Z}_2$-charged D7 branes are unstable. Finally, the non-BPS D-instanton of type I becomes a non-BPS $\mathbb{Z}$-charged brane and it is stable when all radii fulfill $R \geq \sqrt{\alpha'/2}$.

It would be of great interest to extend the techniques used in this article to other type I orbifolds, in particular to those with a phenomenological interest [29], and also to other orientifold models where non-BPS branes may play a more relevant role in the cancellation of tadpoles [17, 18, 30].

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A Ω and the Asymmetric Picture

In this appendix we give a detailed computation of the action of Ω on the boundary states formulated in the asymmetric superghost picture $(-1/2, -3/2)$. The action of Ω on the (untwisted) R-R boundary states in the light-cone gauge formulation was derived before in [22]. However, only the covariant formulation [21] allows the description of D-branes which are of type spacetime-filling or domain-wall with respect to either the bulk or the orbifold fixed planes, hence the relevance of understanding the action of Ω in this formulation. For instance, in the present case, it allows the description of the untwisted R-R sector of a D9-brane and the twisted R-R sector of any D-brane for which all the spatial directions of the orbifold fixed plane are Neumann-directions.

Usually, in order to find out which (BPS) D-branes survive the Ω projection, one may consider the R-R state in the $(-1/2, -1/2)$ picture:

$$|F_{p+2}\rangle = \frac{1}{(p+2)!}\, F_{\mu_0 \ldots \mu_{p+1}} \left(\mathcal{C}_{(10)} \Gamma^{\mu_0 \cdots \mu_{p+1}}\right)_{AB} |A\rangle_{-1/2} \otimes |\widetilde{B}\rangle_{-1/2} \,. \tag{A.1}$$

Applying Ω and taking into account that for the R-R vacuum

$$\Omega \left(|A\rangle_{-1/2} \otimes |\widetilde{B}\rangle_{-1/2}\right) = |\widetilde{A}\rangle_{-1/2} \otimes |B\rangle_{-1/2} = -|B\rangle_{-1/2} \otimes |\widetilde{A}\rangle_{-1/2} \,, \tag{A.2}$$

we find:

$$\Omega\, |F_{p+2}\rangle = -(-1)^{\frac{1}{2}(p+1)(p+2)}\, |F_{p+2}\rangle \,, \tag{A.3}$$

where we have used that $p = odd$ for type IIB and that $\mathcal{C}^T_{(10)} = -\mathcal{C}_{(10)}$. This state is left invariant for $p = 1, 5$, and changes sign for $p = -1, 3, 7$. Notice that here it is crucial that the picture is left-right symmetric, in order to recover the original ordering of the pictures.

On the other hand, the boundary state of the R-R sector in the covariant formulation is constructed upon the vacuum in the asymmetric picture $(-1/2, -3/2)$ [25]. This boundary state is given by[13]

$$|D\rangle_{\mathrm{R,U}} = \frac{1}{2}\left(|D,+\rangle_{\mathrm{R,U}} + |D,-\rangle_{\mathrm{R,U}}\right) , \tag{A.4}$$

with

$$|D,\eta\rangle_{\mathrm{R,U}} = \frac{Q}{2}\,|D_X\rangle|D_\psi,\eta\rangle_{\mathrm{R,U}}|D_{gh}\rangle|D_{sgh},\eta\rangle_{\mathrm{R}} , \tag{A.5}$$

and

$$\begin{aligned} |D_{sgh},\eta\rangle^{(0)}_{\mathrm{R}}|D_\psi,\eta\rangle^{(0)}_{\mathrm{R,U}} &= \mathrm{e}^{i\eta\gamma_0\tilde{\beta}_0}\mathcal{M}_{AB}\,|A\rangle_{-1/2}\otimes\widetilde{|B\rangle}_{-3/2} , \\ \mathcal{M}_{AB}(\eta) &= \left(\mathcal{C}_{(10)}\Gamma^0\cdots\Gamma^p\frac{1+i\eta\Gamma_{11}}{1+i\eta}\right)_{AB} , \qquad p = r+s . \end{aligned} \tag{A.6}$$

However, under the action of Ω, the superghost pictures are interchanged:

$$\Omega\left(|A\rangle_{-1/2}\otimes\widetilde{|B\rangle}_{-3/2}\right) = \widetilde{|A\rangle}_{-1/2}\otimes|B\rangle_{-3/2} = -|B\rangle_{-3/2}\otimes\widetilde{|A\rangle}_{-1/2} . \tag{A.7}$$

In this case, in order to bring it back to the $(-1/2, -3/2)$ picture, a more careful treatment is needed. In order to understand how Ω acts in the states with asymmetric picture we introduce the formal operators [31]

$$\delta(\gamma_m) , \qquad \delta(\beta_n) , \tag{A.8}$$

for each of the holomorphic sectors, which allow to relate different pictures

$$\delta(\beta_{-q-3/2})|q\rangle = |q+1\rangle , \qquad \delta(\gamma_{q+1/2})|q\rangle = |q-1\rangle . \tag{A.9}$$

Moreover, they have the following commutation relations:

$$\begin{aligned} [\beta_m, \delta(\beta_n)] = 0 \quad &, \qquad [\gamma_m, \delta(\beta_n)] = \delta_{n,-m}\frac{d}{d\beta_n}\delta(\beta_n) , \\ [\gamma_m, \delta(\gamma_n)] = 0 \quad &, \qquad [\beta_m, \delta(\gamma_n)] = -\delta_{n,-m}\frac{d}{d\gamma_n}\delta(\gamma_n) . \end{aligned}$$

Therefore, the two possible vacuum states of the superghost zero-modes

$$\beta_0|\downarrow\rangle = 0 , \qquad \gamma_0|\uparrow\rangle = 0 , \tag{A.10}$$

can be characterised as follows:

$$\delta(\beta_0)|\uparrow\rangle = |\downarrow\rangle , \qquad \delta(\gamma_0)|\downarrow\rangle = |\uparrow\rangle , \tag{A.11}$$

[13] For simplicity, we omit the label (r, s) in what follows.

in analogy with the two vacuum states of the ghost zero-modes. On the other hand, considering the overlapping conditions for the zero-mode of the superghost boundary state

$$(\gamma_0 + i\eta\tilde{\gamma}_0)|D_{sgh},\eta\rangle_{\mathrm{R}}^{(0)} = 0\,, \qquad (\beta_0 + i\eta\tilde{\beta}_0)|D_{sgh},\eta\rangle_{\mathrm{R}}^{(0)} = 0\,, \tag{A.12}$$

we can find the following solution to these conditions

$$|D_{sgh},\eta\rangle_{\mathrm{R}}^{(0)} = \delta(\gamma_0 + i\eta\tilde{\gamma}_0)\,|{\downarrow}\rangle \otimes \widetilde{|{\downarrow}\rangle}\,. \tag{A.13}$$

Using the above results, we can rewrite the zero-mode part of the superghost boundary state in the asymmetric $(-1/2,-3/2)$ picture as follows:

$$\begin{aligned} |D_{sgh},\eta\rangle_{\mathrm{R}}^{(0)} &= \mathrm{e}^{i\eta\gamma_0\tilde{\beta}_0}\,|-1/2\rangle \otimes |\widetilde{-3/2}\rangle \\ &= \mathrm{e}^{i\eta\gamma_0\tilde{\beta}_0}\,\delta(\tilde{\gamma}_0)\,|-1/2\rangle \otimes |\widetilde{-1/2}\rangle \\ &= \delta(\tilde{\gamma}_0 - i\eta\,\gamma_0)\,|-1/2\rangle \otimes |\widetilde{-1/2}\rangle\,, \end{aligned} \tag{A.14}$$

which allows to easily derive its transformation rule under Ω:

$$\begin{aligned} \Omega|D_{sgh},\eta\rangle_{\mathrm{R}}^{(0)} &= \delta(\gamma_0 - i\eta\tilde{\gamma}_0)\,|\widetilde{-1/2}\rangle \otimes |-1/2\rangle \\ &= -i\eta\,\delta(\tilde{\gamma}_0 + i\eta\gamma_0)\,|-1/2\rangle \otimes |\widetilde{-1/2}\rangle \\ &= -i\eta\,|D_{sgh},-\eta\rangle_{\mathrm{R}}^{(0)}\,. \end{aligned} \tag{A.15}$$

Furthermore, for the complete superghost state

$$|D_{sgh},\eta\rangle_{\mathrm{R}} = \exp\left(i\eta\sum_{n=1}^{\infty}(\gamma_{-n}\tilde{\beta}_{-n} - \beta_{-n}\tilde{\gamma}_{-n})\right)\,|D_{sgh},\eta\rangle_{\mathrm{R}}^{(0)}\,, \tag{A.16}$$

using that Ω exchange left and right superghost oscillators, we finally obtain:

$$\Omega|D_{sgh},\eta\rangle_{\mathrm{R}} = -i\eta|D_{sgh},-\eta\rangle_{\mathrm{R}}\,. \tag{A.17}$$

Using the above result we can also derive the Ω-projection on the D-branes by using the form of the R-R string state in the $(-1/2,-3/2)$ picture:

$$\begin{aligned} |C^{(p+1)}\rangle &= \frac{1}{2(p+1)!\sqrt{2}}\,C^{(p+1)}_{\mu_1\ldots\mu_{p+1}}\left(\left(\mathcal{C}_{(10)}\Gamma^{\mu_1\ldots\mu_{p+1}}\Pi_+\right)_{AB}\cos\gamma_0\tilde{\beta}_0\right. \\ &\quad \left. +\left(\mathcal{C}_{(10)}\Gamma^{\mu_1\ldots\mu_{p+1}}\Pi_-\right)_{AB}\sin\gamma_0\tilde{\beta}_0\right)\,|A\rangle_{-1/2}\otimes\widetilde{|B\rangle}_{-3/2}\,. \end{aligned} \tag{A.18}$$

This analysis is very useful for checking whether there are spacetime-filling D-branes in the spectrum. We have derived above that for the zero modes:

$$\Omega\left(\mathrm{e}^{i\eta\gamma_0\tilde{\beta}_0}|A\rangle_{-1/2}\otimes\widetilde{|B\rangle}_{-3/2}\right) = i\eta\,\mathrm{e}^{-i\eta\gamma_0\tilde{\beta}_0}\,|B\rangle_{-1/2}\otimes\widetilde{|A\rangle}_{-3/2}\,, \tag{A.19}$$

which implies

$$\begin{aligned} \Omega\left(\cos\gamma_0\tilde{\beta}_0\,|A\rangle_{-1/2}\otimes\widetilde{|B\rangle}_{-3/2}\right) &= \sin\gamma_0\tilde{\beta}_0\,|B\rangle_{-1/2}\otimes\widetilde{|A\rangle}_{-3/2}\,, \\ \Omega\left(\sin\gamma_0\tilde{\beta}_0\,|A\rangle_{-1/2}\otimes\widetilde{|B\rangle}_{-3/2}\right) &= \cos\gamma_0\tilde{\beta}_0\,|B\rangle_{-1/2}\otimes\widetilde{|A\rangle}_{-3/2}\,. \end{aligned} \tag{A.20}$$

Finally, using this result and the properties of the Γ-matrices we find:

$$\Omega\,|C^{(p+1)}\rangle = -(-1)^{\frac{1}{2}p(p+1)}\,|C^{(p+1)}\rangle\,, \tag{A.21}$$

hence it is invariant for $p = 1, 5, 9$, as seen before.

Likewise, we can find the action of Ω on the quantum state of the R-R twisted sector in the $(-1/2, -3/2)$ picture:

$$\begin{aligned} |C^{(r+1)}\rangle_{\mathrm{T}} \;=\; & \frac{1}{\sqrt{2}(p+1)!} C^{(r+1)}_{\mu_1\ldots\mu_{r+1}}\, \left(\left(\mathcal{C}_{(6)}\gamma^{\mu_1\ldots\mu_{p+1}}\Pi_+\right)_{ab}\,\cos\gamma_0\tilde{\beta}_0\right. \\ & \left. +\;\left(\mathcal{C}_{(6)}\gamma^{\mu_1\ldots\mu_{p+1}}\Pi_-\right)_{ab}\,\sin\gamma_0\tilde{\beta}_0\right)\,|a\rangle_{-1/2,\mathrm{T}}\otimes\widetilde{|b\rangle}_{-3/2,\mathrm{T}}\,. \end{aligned} \tag{A.22}$$

Using (A.20) and the properties of the 6-dimensional γ-matrices we find:

$$\Omega\,|C^{(r+1)}\rangle_{\mathrm{T}} = (-1)^{\frac{1}{2}r(r+1)}\,|C^{(r+1)}\rangle_{\mathrm{T}}\,, \tag{A.23}$$

hence it is invariant for $r = -1$, and 3.

B Action of Ω on the Twisted NS-NS vacuum

In this Appendix we give an explicit analysis of the action of Ω in the twisted NS-NS vacuum. This vacuum state consists of the spin fields constructed out of the four fermionic zero modes in the left and four from the right moving sector. The superghost vacuum is taken to be in the $(-1,-1)$ picture. In each of the left and right sector, these fermion zero modes satisfy an $SO(4)$ Clifford algebra. Thus we have

$$\{\psi_0^i, \psi_0^j\} = \delta^{ij} = \{\tilde{\psi}_0^i, \tilde{\psi}_0^j\}\,, \qquad \{\psi_0^i, \tilde{\psi}_0^j\} = 0. \tag{B.1}$$

where i, j take values from 6,7,8 and 9. Our conventions for the $SO(4)$ gamma-matrices are as follows:

$$\{\bar{\gamma}^i, \bar{\gamma}^j\} = 2\delta^{ij}\,, \qquad \mathcal{C}_{(4)}^T = -\mathcal{C}_{(4)}\,, \qquad [\mathcal{C}_{(4)}, \bar{\gamma}] = 0\,, \qquad (\bar{\gamma}^i)^T = -\mathcal{C}_{(4)}\bar{\gamma}^i\mathcal{C}_{(4)}^{-1}\,, \tag{B.2}$$

with $\bar{\gamma} = -\bar{\gamma}^6\bar{\gamma}^7\bar{\gamma}^8\bar{\gamma}^9$. Let α, β denote the spinor indices in four dimensions and let $|\alpha\rangle_{\mathrm{T}}\otimes\widetilde{|\beta\rangle}_{\mathrm{T}}$ denote the twisted spinor vacuum constructed from the spin fields of the above fermionic zero modes. The action of the fermionic oscillators and zero modes in this basis is defined to be

$$\begin{aligned} \psi_n^i|\alpha\rangle_{\mathrm{T}}\otimes\widetilde{|\beta\rangle}_{\mathrm{T}} &= \tilde{\psi}_n^i|\alpha\rangle_{\mathrm{T}}\otimes\widetilde{|\beta\rangle}_{\mathrm{T}} = 0\,, \qquad \forall n\geq 1\,, \\ \psi_0^i|\alpha\rangle_{\mathrm{T}}\otimes\widetilde{|\beta\rangle}_{\mathrm{T}} &= \frac{1}{\sqrt{2}}\left(\bar{\gamma}^i\right)^{\alpha}{}_{\delta}\,(1\!\!1)^{\beta}{}_{\rho}\,|\delta\rangle_{\mathrm{T}}\otimes\widetilde{|\rho\rangle}_{\mathrm{T}}\,, \\ \tilde{\psi}_0^i|\alpha\rangle_{\mathrm{T}}\otimes\widetilde{|\beta\rangle}_{\mathrm{T}} &= \frac{1}{\sqrt{2}}\,(\bar{\gamma})^{\alpha}{}_{\delta}\left(\bar{\gamma}^i\right)^{\beta}{}_{\rho}\,|\delta\rangle_{\mathrm{T}}\otimes\widetilde{|\rho\rangle}_{\mathrm{T}}\,. \end{aligned} \tag{B.3}$$

As in the earlier cases, the action of Ω is to interchange the left and right moving sectors. Thus the defining relation for the action of Ω can be obtained by demanding simply that, namely

$$\Omega\left(\tilde{\psi}_0^i|\alpha\rangle_{\mathrm{T}}\otimes\widetilde{|\beta\rangle}_{\mathrm{T}}\right)=\psi_0^i\widetilde{|\alpha\rangle}_{\mathrm{T}}\otimes|\beta\rangle_{\mathrm{T}}\,,\tag{B.4}$$

or equivalently

$$\Omega\left(\psi_0^i|\alpha\rangle_{\mathrm{T}}\otimes\widetilde{|\beta\rangle}_{\mathrm{T}}\right)=\tilde{\psi}_0^i\widetilde{|\alpha\rangle}_{\mathrm{T}}\otimes|\beta\rangle_{\mathrm{T}}\,,\tag{B.5}$$

where

$$\widetilde{|\alpha\rangle}_{\mathrm{T}}\otimes|\beta\rangle_{\mathrm{T}}\;=-\,|\beta\rangle_{\mathrm{T}}\otimes\widetilde{|\alpha\rangle}_{\mathrm{T}}\tag{B.6}$$

These definitions guarantee that $\Omega^2=1$. Using the equations in (B.3), the first equation (B.4) gives:

$$(\bar{\gamma})^{\alpha}{}_{\delta}\,\left(\bar{\gamma}^i\right)^{\beta}{}_{\rho}\,\Omega\left(|\delta\rangle_{\mathrm{T}}\otimes\widetilde{|\rho\rangle}_{\mathrm{T}}\right)=-\,\left(\bar{\gamma}^i\right)^{\beta}{}_{\rho}\,(1\!\!1)^{\alpha}{}_{\delta}\,|\rho\rangle_{\mathrm{T}}\otimes\widetilde{|\delta\rangle}_{\mathrm{T}}\,,\tag{B.7}$$

which can be simplified to obtain

$$\Omega\left(|\alpha\rangle_{\mathrm{T}}\otimes\widetilde{|\beta\rangle}_{\mathrm{T}}\right)=-\,(\bar{\gamma})^{\alpha}{}_{\delta}\,|\beta\rangle_{\mathrm{T}}\otimes\widetilde{|\delta\rangle}_{\mathrm{T}}\,.\tag{B.8}$$

Similarly, the equation (B.5) gives rise to

$$\left(\bar{\gamma}^i\right)^{\alpha}{}_{\delta}\,(1\!\!1)^{\beta}{}_{\rho}\,\Omega\left(|\delta\rangle_{\mathrm{T}}\otimes\widetilde{|\rho\rangle}_{\mathrm{T}}\right)=-\,(\bar{\gamma})^{\beta}{}_{\rho}\,\left(\bar{\gamma}^i\right)^{\alpha}{}_{\delta}\,|\rho\rangle_{\mathrm{T}}\otimes\widetilde{|\delta\rangle}_{\mathrm{T}}\,,\tag{B.9}$$

which can be simplified to obtain

$$\Omega\left(|\alpha\rangle_{\mathrm{T}}\otimes\widetilde{|\beta\rangle}_{\mathrm{T}}\right)=-\,(\bar{\gamma})^{\beta}{}_{\delta}\,|\delta\rangle_{\mathrm{T}}\otimes\widetilde{|\alpha\rangle}_{\mathrm{T}}\,.\tag{B.10}$$

If we denote the superghost vacuum in the $(-1,-1)$ picture as $|-1\rangle\otimes\widetilde{|-1\rangle}$, with

$$\Omega\left(|-1\rangle\otimes\widetilde{|-1\rangle}\right)=\widetilde{|-1\rangle}\otimes|-1\rangle=-|-1\rangle\otimes\widetilde{|-1\rangle}\,,\tag{B.11}$$

and noting that the full vacuum is given by

$$|\alpha\rangle_{-1,\mathrm{T}}\otimes\widetilde{|\beta\rangle}_{-1,\mathrm{T}}=|\alpha\rangle_{\mathrm{T}}|-1\rangle\otimes\widetilde{|\beta\rangle}_{\mathrm{T}}\widetilde{|-1\rangle}\,,\tag{B.12}$$

we can write the action of Ω on the full vacuum of the twisted NS-NS sector as follows:

$$\Omega\left(|\alpha\rangle_{-1,\mathrm{T}}\otimes\widetilde{|\beta\rangle}_{-1,\mathrm{T}}\right)=(\bar{\gamma})^{\alpha}{}_{\delta}\,|\beta\rangle_{-1,\mathrm{T}}\otimes\widetilde{|\delta\rangle}_{-1,\mathrm{T}}=(\bar{\gamma})^{\beta}{}_{\delta}\,|\delta\rangle_{-1,\mathrm{T}}\otimes\widetilde{|\alpha\rangle}_{-1,\mathrm{T}}\,,\tag{B.13}$$

as previously announced in (2.34). Finally, note that this relation is equivalent to

$$|\alpha\rangle_{-1,\mathrm{T}}\otimes\widetilde{|\beta\rangle}_{-1,\mathrm{T}}=(\bar{\gamma})^{\alpha}{}_{\delta}\,(\bar{\gamma})^{\beta}{}_{\rho}\,|\delta\rangle_{-1,\mathrm{T}}\otimes\widetilde{|\rho\rangle}_{-1,\mathrm{T}}\,,\tag{B.14}$$

which is consistent since both left and right spinors are of the same chirality.

C Crosscaps in Type I on $T^4/\mathbb{Z}_2$

The tadpole-cancelling D9 and D5 branes in type I on $T^4/\mathcal{I}_4$ are bulk branes. In type I on a K3 orbifold we have a 9-crosscap and 5-crosscaps which have boundary states in the untwisted sectors only:

$$|C9\rangle = |C9\rangle_{\rm NS} + |C9\rangle_{\rm R}\,, \qquad |C5\rangle = |C5\rangle_{\rm NS} + |C5\rangle_{\rm R}\,. \tag{C.1}$$

As usual, each part is defined in terms of the spin structures:

$$\begin{aligned} |C\rangle_{\rm NS} &= \frac{1}{2}\left(|C,+\rangle_{\rm NS} - |C,-\rangle_{\rm NS}\right)\,, \\ |C\rangle_{\rm R} &= \frac{1}{2}\left(|C,+\rangle_{\rm R} + |C,-\rangle_{\rm R}\right)\,, \end{aligned} \tag{C.2}$$

where

$$|C,\eta\rangle_{\rm NS/R} = |C_X\rangle |C_\psi,\eta\rangle_{\rm NS/R} |C_{gh}\rangle |C_{sgh},\eta\rangle_{\rm NS/R}\,. \tag{C.3}$$

This generic form is common to both crosscaps:

$$\begin{aligned} |C_\psi,\eta\rangle_{\rm NS} &= \prod_{r=1/2}^{\infty} {\rm e}^{i\eta(-1)^r \psi_{-r}\cdot S\cdot\tilde\psi_{-r}}\,|0\rangle\,, \\ |C_{sgh},\eta\rangle_{\rm NS} &= \prod_{r=1/2}^{\infty} {\rm e}^{i\eta(-1)^r(\gamma_{-r}\tilde\beta_{-r}-\beta_{-r}\tilde\gamma_{-r})}\,|{-1}\rangle\otimes|\widetilde{-1}\rangle\,, \\ |C_{gh}\rangle &= \prod_{n=1}^{\infty} {\rm e}^{(-1)^n(c_{-n}\tilde b_{-n}-b_{-n}\tilde c_{-n})}\left(\frac{c_0+\tilde c_0}{2}\right)|1\rangle\otimes|\widetilde{1}\rangle\,, \\ |C_\psi,\eta\rangle_{\rm R} &= \prod_{m=1}^{\infty} {\rm e}^{i\eta(-1)^m \psi_{-m}\cdot S\cdot\tilde\psi_{-m}}\,|C_\psi,\eta\rangle_{\rm R}^{(0)}\,, \\ |C_{sgh},\eta\rangle_{\rm R} &= \prod_{m=1}^{\infty} {\rm e}^{i\eta(-1)^m(\gamma_{-m}\tilde\beta_{-m}-\beta_{-m}\tilde\gamma_{-m})}\,|C_{sgh},\eta\rangle_{\rm R}^{(0)}\,, \end{aligned} \tag{C.4}$$

with $S_{\mu\nu} = \eta_{\mu\nu}$ for the 9-crosscap and $S_{\mu\nu} = (\eta_{\alpha\beta}, -\delta_{ij})$, for the 5-crosscap, where $\alpha,\beta = 0,\ldots,5$ are the directions on the fixed planes and $i,j = 6,\ldots,9$ are the directions along T^4. The zero modes in the R-R sector are given by:

$$\begin{aligned} |C_\psi,\eta\rangle_{\rm R}^{(0)} &= \left(\mathcal{C}_{(10)}\Gamma^0\cdots\Gamma^p\frac{1+i\eta\Gamma_{11}}{1+i\eta}\right)_{AB}|A\rangle\otimes|\widetilde{B}\rangle\,, \qquad p=5,9\,, \\ |C_{sgh},\eta\rangle_{\rm R}^{(0)} &= {\rm e}^{i\eta\gamma_0\tilde\beta_0}\,|{-1/2}\rangle\otimes|\widetilde{-3/2}\rangle\,, \end{aligned} \tag{C.5}$$

The fact that the orbifold is compact is reflected in the bosonic-oscillator part of the crosscaps states. The 9-crosscap contains windings along the T^4:

$$|C9_X\rangle = \mathcal{N}_9\left(\sum_{m\in\mathbb{Z}} {\rm e}^{iq_m\frac{mR}{\alpha'}}\right)^4 \prod_{n=1}^{\infty} {\rm e}^{-\frac{1}{n}(-1)^n\alpha_{-n}\cdot S\cdot\tilde\alpha_n}\,|0\rangle\,, \tag{C.6}$$

whereas the 5-crosscap contains momentum modes along the T^4 directions:

$$|C5_X\rangle = \mathcal{N}_5 \left(\sum_{n\in\mathbb{Z}} \mathrm{e}^{iq_n \frac{n}{R}} \right)^4 \prod_{n=1}^{\infty} \mathrm{e}^{-\frac{1}{n}(-1)^n \alpha_{-n}\cdot S\cdot\tilde{\alpha}_n} |0\rangle \,. \qquad \text{(C.7)}$$

For simplicity we have taken all four radii equal to each other.

The normalisation of these crosscaps states can be derived by comparing the open string Möbius amplitudes with a closed string calculation:

$$\begin{aligned} \mathcal{N}_9 &= \frac{1}{\sqrt{2}} 2^5 \frac{T_9}{2} \left(\frac{2\pi R}{\Phi} \right)^2 \,, \\ \mathcal{N}_5 &= \frac{1}{\sqrt{2}} 2^5 \frac{T_5}{2} \left(\frac{2\pi R}{\Phi} \right)^2 (2\pi R)^{-4} \,, \\ T_p &= \sqrt{\pi}(2\pi\sqrt{\alpha'})^{3-p} \,, \qquad p = r + s \,. \end{aligned}$$

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